QUICK BLOCKS:
Developing Spatial Sense

Videos and classroom dialogue show the importance of giving K–12 students opportunities to engage in spatial reasoning and number sense activities.

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Have you ever noticed that some students seem to be able to visualize three-dimensional (3D) objects or situations within problems better than others? Through our teaching experiences, we have observed that students' spatial sense improved as they worked with 3D manipulatives during problem solving. Encouraged by these teaching experiences, we set out to create a short classroom routine that our students would find challenging and enjoyable and that would allow opportunities to develop spatial reasoning as well as number sense. This article explores the importance of giving students time to develop mental imagery in mathematics, especially 3D imagery involving cubes. We give a description of the development process we conducted to design the Quick Block images, and we share our experiences of engaging students in the Quick Blocks activity. We also give free access to our electronic files to be used by teachers with their students at all levels.

THE IMPORTANCE OF SPATIAL SENSE

Spatial sense has been an important area of interest for mathematics educators across several decades. In 1989, NCTM's *Curriculum and Evaluation Standards for School Mathematics* put spatial understanding in clear view for consideration by all mathematics teachers by stating that "spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world" (p. 48). In 1990, *The Arithmetic Teacher* journal published a Focus Issue on spatial sense, stating that "the development of spatial sense is vital for learners of all ages" (Shaw, p. 5). The ability to visualize and imagine mathematical ideas through mental imagery has come to be understood as not only something mathematicians do (Sfard 1994) but also an important part of mathematical reasoning (Brown and Wheatley 1997). As children engage in mathematical activity, they use imagery to make sense of significant mathematics (Reynolds 1993). The majority of preschool students use mental models to solve arithmetic problems, and this requires the development and use of their visual-spatial working memory (Rasmussen and Bisanz 2005). Furthermore, the development of students' spatial sense is important to their mathematics success (Battista 2001; Lee, Lee, and Collins 2010; Reynolds and Wheatley 1997; Thompson 2016).

In our teaching practice, we have found that tasks involving spatial reasoning afford students of different perspectives and strengths to join the class's mathematical discussions. We consider such access to mathematics an important part of our equitable mathematics teaching practice (NCTM 2014), so when justifiable, we attend to incorporating task elements involving spatial reasoning. Moreover, students' spatial skills are not fixed but rather can grow as they engage in tasks involving spatial reasoning (Uttal et al. 2013). It is therefore important for mathematics teachers to consider tasks that nurture students' spatial sense.

We have noticed that asking our students to reflect on imagery builds their enjoyment during learning and has allowed them to make deeper connections across mathematical ideas (Putri 2019). During the planning of our mathematics lessons, we consider ways in which spatial reasoning can support student engagement in other mathematical topics. One area of importance for us is connecting spatial sense with number sense. Children who develop stronger spatial sense have been found to be more successful in their number sense (Gunderson et al. 2012). Recognizing that children's spatial sense can grow and that it helps strengthen other mathematical areas inspired our hopes and designs of the Quick Blocks activity.

DEVELOPMENT OF QUICK BLOCKS

Our ideas for the development of Quick Blocks draw heavily from Wheatley's *Quick Draw* (2007) and number sense ideas found in Wheatley and Reynolds' *Coming to Know Number* (2010). Quick Blocks was built on the basis of three other classroom routines: Quick Draw, Quick Images, and Quick Build.

Classroom Routines for Spatial and Number Sense

As teachers, we have long found benefit in the use of Quick Draw as a routine to develop our students' spatial sense. During a Quick Draw episode, the teacher briefly shows students an image (see figure 1) containing arrangements or pieces of geometric figures and then hides it. Students then draw what they remember seeing. The image is briefly shown a second time, and some students choose to redraw it more precisely or finish their original drawing. Following this, the

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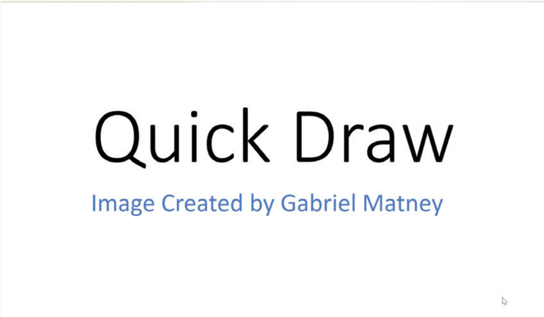
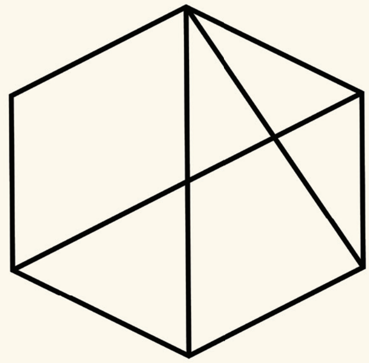
Video 1 Experiencing a Quick Draw Session



Watch the full video online.

Fig. 1



Example of a Quick Draw image to engage students in spatial reasoning

students and the teacher converse as a class about what they saw and how they drew it. We have found that this activity opens our students' mathematical minds at the beginning of class. It also improves their ability to sketch and draw, develops their spatial sense, and awakens their noticing of mathematical relationships among geometric figures. To engage in a Quick Draw experience, gather a piece of paper and a writing utensil while watching video 1.

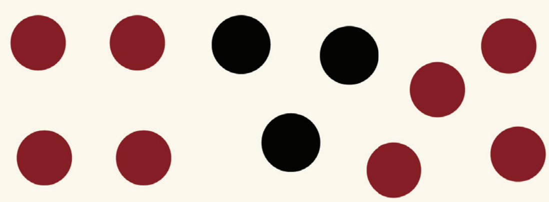
Video 2 offers a glimpse into Shannon Gladieux's fourth-grade classroom as students engage in a Quick Draw episode. The video, which is less than two minutes long, shows students verbalizing different views of the mathematical objects and exploring some academic language involving the geometry seen in the image.

Another type of classroom routine that we use, called Quick Images, promotes subitizing, the ability to know instantaneously how many objects are there (Clements 1999, p. 402). Subitizing is fundamental to the development of other components of number sense, such as unitizing, counting strategies, composing, and decomposing. Quick Images is similar to Quick Draw in that images of dots in various arrangements and quantities (see figure 2) are shown only briefly to students before the image is taken away from view. Students are not shown the image long enough to be able to count one by one. After the image is taken away, students are challenged to consider how many dots

Video 2 Shannon Gladieux Engages Students in Quick Draw

Watch the full video online.

Fig. 2



Example of a Quick Image given to students to develop number sense

Video 3 Experiencing a Quick Image Session

Watch the full video online.

they saw and explain how they determined the number of dots. Students see and form ideas about the number of dots in different ways, so as a whole class, students share their perspectives on efficient ways to compose and decompose the number of dots seen.

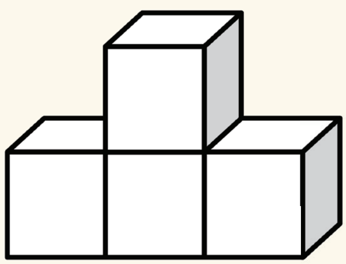
In video 3, we share how Quick Images can differ in arrangement and color, and we explore the reasons teachers might have for showing a particular image. Video 3 also provides an example engagement in Quick Images for those who would like to better understand what it means to instantiate this routine in the classroom.

Quick Draw and Quick Images engage students in important mental mathematics experiences. We continually seek to give our students a variety of classroom routines that allow for productive struggle (NCTM 2014). Having seen that our students' spatial sense benefited from kinesthetic engagement with manipulatives during problem solving, we sought a task that would promote mental mathematics but also would allow for kinesthetic construction. We found the groundwork for such a task in Wheatley and Reynolds' *Coming to Know Number* (2010), in which they briefly describe and share six examples of a task named Quick Build. Similar to the previous classroom routines, students are briefly shown images of 3D cubes (see figure 3) and then asked to construct the image with manipulative blocks that connect. Students discuss what they saw and explain how they built what they saw. We leverage each of these routines to help foster norms for mathematical discourse and for promoting students' construction of viable arguments and ability to critique the reasoning of others (NGA Center and CCSSO 2010). We used our knowledge and experience with these three classroom routines to design the Quick Blocks images.

Designing Images to Connect Spatial and Number Sense

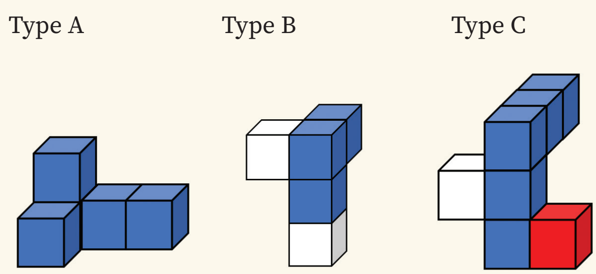
We set out to build on the image templates from Quick Build to connect the development of spatial sense with the subitizing experiences of Quick Images. To do this, we began to create our own 3D cubic images, incorporating different colors, shapes, and orientations. We created images of three different types (see figure 4). The types represent the use of color in the image with Types A, B, and C representing one, two, or three

Fig. 3



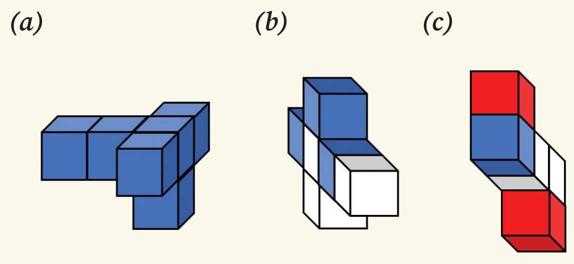
Example of a Quick Build image

Fig. 4



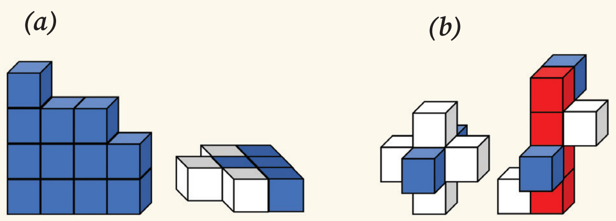
Quick Block Type A, Type B, and Type C are differentiated by the number of colors in the image.

Fig. 5



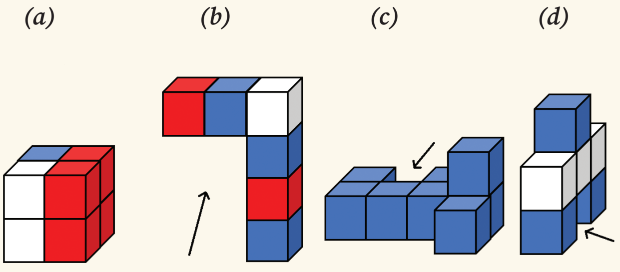
Examples of the types of Quick Block viewing orientations: (a) from above-right, (b) from above-left, and (c) from below-right

Fig. 6



Examples of the types of (a) 2D organization and (b) 3D organization found in Quick Blocks

Fig. 7



Examples of the different types of negative space demarcations in Quick Blocks: (a) no negative space, (b) negative space demarcated by two sides, and (c, d) negative space demarcated by three sides

colors, respectively. To challenge our students' sense of visual orientation, we designed images from different points of view (see figure 5). Next, we wanted our students to experience images in which the cubes might be said to form a single plane, which we refer to as 2D organization, versus situations in which the structure of the cubes depicts 3D organization (see figure 6). Our last consideration was the creation of figures that demarcated negative space in different ways. Figure 7a does not demarcate any negative space; the cubes are all tightly packed. Figure 7b demarcates a negative space with two sides, whereas figures 7c and 7d both demarcate a negative space with three sides.

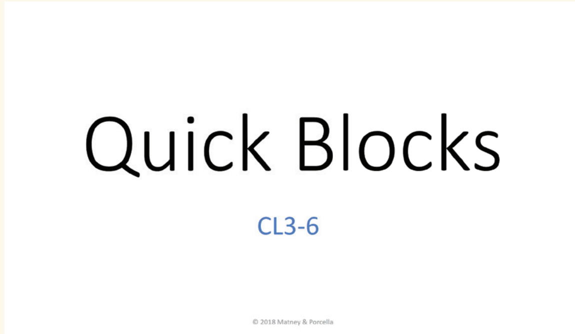
When engaging students in Quick Blocks, we follow the convention established by Wheatley (2007) whereby students will have at least two opportunities to view the image. The convention is also seen in the videos accompanying this article. The teacher's cue for students to look up at the image is to begin gently counting down—three, two, one—and then briefly reveal the image before concealing it. Students then begin their attempts to physically construct what they saw. As students construct, the teacher observes, noting the ways students are constructing various images and whether the construction matches the image. The teacher then gently counts down a second time—three, two, one—and again briefly shows the image before concealing it. After the second look, students often complete or modify their construction, twisting and lifting it to match the orientation of the image as they seek to ensure the correctness of their construction. The class discusses what students saw initially and how they went about reconstructing the image with the blocks. The teacher asks students to share different observations and to consider the extent to which the observation was helpful. Students discuss what they saw and explain differences in their construction. The teacher reveals the image and allows students to inspect their work. Finally, students suggest new ideas, including efficient and effective strategies to view and subitize the image for successful reconstruction. Teachers can experience a Quick Blocks task by gathering red, white, and blue blocks and watching video 4. Videos 5 and 6 show Julia Porcella's ninth graders and Gladieux's fourth-grade students working on Quick Blocks tasks.

The design of each image incorporates the elements of shape, pattern, and color so that students have a variety of spatial pathways to envision the number of blocks and their orientation to one another. All of our developed Quick Block images can be seen in the Quick Block images file accompanying this article (see the appendix, available as an online supplemental file). As videos 5 and 6 and the student scenarios that follow show, students connect different spatial and numeric elements to successfully reconstruct the blocks physically. When teachers provide students with multiple pathways to visualize geometric structures, they increase accessibility for all students (NCTM 2014). In our classrooms, we also seek to nurture a norm of finding more than one way to justify a solution or our thinking about a problem. Such tasks as Quick Blocks—that have multiple pathways through which a student might be successful—open opportunities for students to engage in this norm and to creatively explore the many possible perspectives before any prompting by the teacher.

As students engage in the Quick Blocks task, we want them to be both challenged and successful. Allowing students two opportunities to construct the cubic image is challenging because students naturally want to complete it, if possible, after the first viewing. However, if they cannot complete it or are not yet secure in their result, students know they have a second viewing opportunity. This provides for an enjoyable experience in productive struggle because students have the time for self-correction, self-monitoring, and self-regulation of improvement (NCTM 2014). We noticed that some images were far more likely for students to get correct on the first attempt. We ran trials of each Quick Block image with several classes of fourth-grade students to determine which ones were more difficult. To assist other teachers in knowing which images tended to be more difficult, we organized them into levels: Level 1 images are those most students get correct on the first try; levels 2 and 3 indicate increased difficulty for students. Level 1 images nearly all contain a 2D organization and a small number of cubes. Level 2 images are mostly 3D organizations, contain more cubes than level 1, and have some images demarcating negative space. Level 3 images contain more complex 3D organizations, often contain more cubes than level 1 or 2 images, and some level 3 images demarcate more than one negative space. Each of the three levels contains images consisting of blocks with one, two, and three colors and orientations.

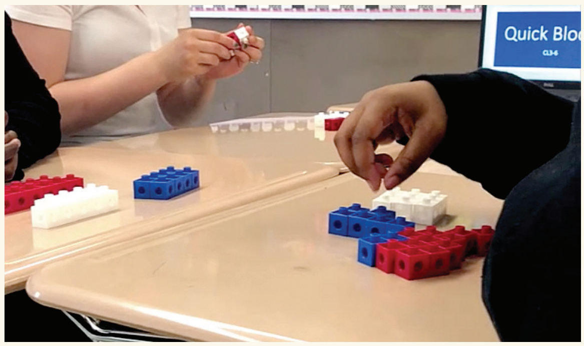
Quick Blocks activities add a kinesthetic element that is not present in Quick Draw and Quick Images tasks. As students process the image, they must mentally represent it and then physically construct it. During the process of construction, students notice that the orientation of the blocks, as they lie on the desk, does not exactly match the orientation of the images. Students must manipulate the blocks, through physical rotation and

Video 4 Julia Porcella's Ninth Graders Engage in Quick Blocks



Watch the full video online.

Video 5 Quick Blocks Session Explanation



Watch the full video online.

Video 6 Fourth Graders Engage with Two Different Images

Watch the full video online.

Table 1 Modifications When Enacting Quick Blocks Routines

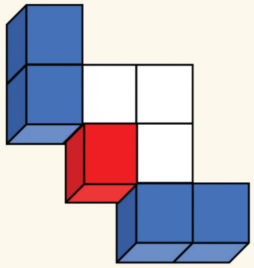
Grade Band	Top 10 Favorites	Modifications
PK–2	AL1-1 AL1-2 AL1-3 AL1-5 AL1-9 AL2-3 BL1-1 BL1-2 CL1-1 CL1-2	Students in this grade band may benefit from initial experiences building with the blocks without the time constraint. Students at this grade level may benefit from extended viewing time of Quick Block images or building with the image left on the screen. Conversations about patterns observed or subitizing will be helpful in scaffolding students toward success with this routine.
3–5	AL1-2 AL1-7 AL2-5 AL2-8 BL1-9 BL2-2 BL3-6 CL1-3 CL3-8 CL3-9	Students in this grade band may benefit from limited introductory experiences building with the blocks without the time constraint (allowing the image to stay on the screen during build time). Extended viewing time can be provided on the basis of students' needs, but the image should still flash and disappear once students are familiar with the routine. Mathematical conversations focusing on what they saw and how they saw it can be helpful with the image still showing when teachers are beginning to develop this thinking routine with students. Once students develop the necessary vocabulary, switch to using the standard method.
6–8	AL1-6 AL1-8 AL2-7 AL3-5 BL1-5 BL2-1 BL2-5 CL1-5 CL2-6 CL3-7	Students in this grade band may benefit from initial experiences with level 1 figures before moving on to more advanced figures. Conversation should focus on the building process and on seeing structure in level 1 figures before moving to more complex figures, allowing students to begin developing pattern recognition.
9–12	AL1-8 AL2-7 AL3-4 BL1-10 BL2-4 BL2-9 CL1-3 CL2-3 CL2-10 CL3-6	Students in this grade band may benefit from the challenge of more complex figures after success with level 1 figures. Sufficient time for conversation and student explanation is still required at this grade level to develop spatial sense. Students should be encouraged to use mathematical language and vocabulary when describing their process of building the figures.

Also see the appendix, our top 10 images for each grade band.

translation, to find an alignment between their remembered mental imagery and the physical blocks in front of them. They do this while also attending to the colors of the blocks and the way blocks or groups of blocks relate to one another. This is a challenging task, especially the first few times students attempt it. As the images' level of difficulty increases, students are challenged to consider whether or not their physical creation is accurate and are often seen thinking about how to turn the construction so as to represent the perspective of the given image.

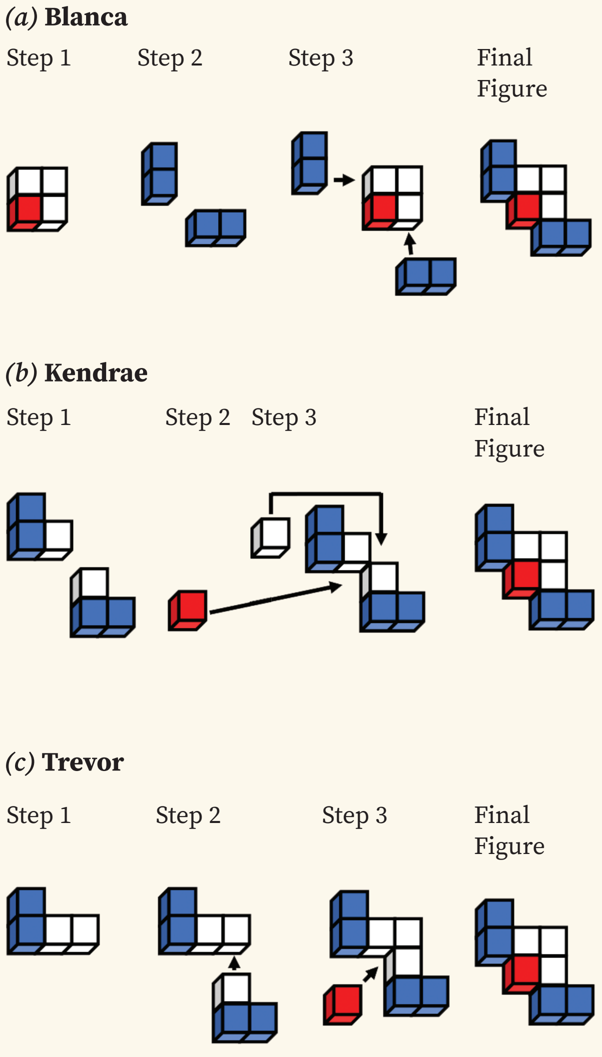
We have used Quick Blocks with students of all ages. Engaging students of different ages requires some thoughtful yet minor modifications on the part of the teacher. In table 1, we share suggested modifications teachers might consider when enacting the Quick Blocks routine with children in various grade bands.

Fig. 8



Original image shown to students

Fig. 9



Demonstration of the steps involved in (a) Blanca's, (b) Kendrae's, and (c) Trevor's thinking

We also share our top 10 favorite images to begin with at each grade band (also see the appendix).

EXPLORING QUICK BLOCKS WITH STUDENTS

The first author, Gabriel Matney, began developing Quick Blocks and ran early trials with students from kindergarten to grade 12. He conducted six trials with small groups of three students from similar grade levels. Children were given a set of 10 Quick Block images. From these trials two things became apparent: (1) Some students had an uncanny ability to connect the image to the construction of the physical object, and (2) all students appeared to improve the more they experienced the activity. For example, one trial had three students: a kindergartner, a second grader, and a third grader. In this trial, the children were all shown 10 Quick Blocks images, one at a time, for three seconds. The image was then hidden, and the students used individual connecting cubes to try to represent the image in physical form. After the children had time to construct the image, they were shown the image for another three seconds and given another opportunity to finalize their construction on the basis of the second look. In nine of the 10 trials, the kindergartner correctly constructed the physical representation the first time. Furthermore, the kindergartner was able to more quickly assemble the blocks for each of the 10 images. Therefore, we note that although students may display a wide variance in ability, success with this task does not depend on age or grade level.

The second-grade student needed two viewings of the image to successfully construct the first six images. Similarly, the third grader needed two viewings of the first seven images to be successful. In other words, the second- and third-grade students were unable to successfully construct the image with blocks after the first viewing until they had done several Quick Block images. As they learned and explored the relationship between the cubes and the images, their ability to orient and assemble the cubes according to the image improved. During the last several years, we have shared Quick Blocks with mathematics teaching colleagues at all levels—elementary, middle, high school, and university preservice teachers. Across these cases, we noticed sets of students who find early success with Quick Blocks much as the kindergartner did. Similarly, much as the second- and third-grade students, all students became more successful as they gained experience with constructing cubic imagery and discussing their visualization of the image with one another.

One Image, Diverse Ways of Seeing

We share one excerpt of a Quick Blocks dialogue to reveal the diversity of ways students construct images. The excerpt also demonstrates the way students have the opportunity to bring in academic language, question one another, and adapt explanations to help others understand. In the following scenario, these sixth-grade students have been working on a unit involving symmetry, reflections, translations, and rotations. The teacher enacted the routine with the image shown in figure 8, giving the students two opportunities to view and construct the image.

Teacher: [*Motioning for the class to come together.*] Blanca has volunteered to share her way, so let's focus on her ideas at this time.

Blanca: At first I saw a square in the middle with three white and one red cube. Then I quickly saw four blue cubes composed two up and down and two side to side, like this. Then I put all three pieces together. Any questions? (See figure 9a.)

Maggie: I also saw that there were four blue, but today the picture was so fast, so I didn't remember, umm, at first, where the blue cubes went. How did you know where to put the blue blocks?

Blanca: It was fast! But from before, when we talked about negative space, I saw the two Ls like in other pictures. So I knew the blue created those [*two negative space Ls*] by branching out from the sides of the square. Any more questions?

Class: [*Shaking their heads no. Kendrae raises his hand, and the teacher motions to him.*]

Kendrae: I also saw Ls, but I saw different Ls than Blanca. I saw two Ls that are made up of two blues and one white. Then those Ls were held together by one red and one white cube, like this. So that is how I did it. Any questions or comments on my way? (See figure 9b.)

Trevor: I also saw that, but I saw a loner L on top with two blue and two white. So I made both Ls and then snapped them together with the one red in the middle. (See figure 9c.)

Kendrae: I like your way too.

Class: [*Motioning to Kendrae and Trevor that they agree with the method as Tameka raises her hand.*]

Tameka: I noticed that Kendrae's Ls are translations of one another. Well, kind of. You see how you can just slide one on top of the other and they match? But then I noticed that the colors do not match. [*Shrugging*] So, are they a translation?

Teacher: Hmmm, Tameka is posing an interesting question here. Does everyone see the two Ls Kendrae and Tameka are talking about? [*Class nods affirmatively.*] OK, let's spend a moment thinking about Tameka's question with our shoulder partners. Are Kendrae's Ls a translation?

Class: [*Pairs of students begin talking and manipulating their cubes. Several pairs of students bring up different rigid transformations during their discussion. The teachers ask two different pairs of students to share their ideas.*]

Emilio/Daria: We think these are not just a translation. But if we start with the bottom L (two blue cubes with a white on top) then we can take the top L, reflect it like this, then rotate it, and then it matches up where the white cube is on top. See?

Arya/Chris: We kind of agree with Emilio and Daria's idea but we have two answers. If you look at both shape and color, then Emilio and Daria are right: It takes more than a translation to match the Ls. But, actually, if you just consider the shape, like if all the blocks were blue, then the Ls are already oriented the same, and they are the same shape, so a translation would show that they matched.

In this scenario, students demonstrated their different ways of conceptually subitizing (Clements 1999). Whether focusing on color (Blanca) or on self-similar pieces (Kendrae), students quickly recognized the number of cubes needed to construct the image. We often see students choosing to subitize in ways that involve the images' shape, pattern, and color (see also videos 5 and 6). Depending on the image, students discuss both additive and multiplicative ideas related to what is needed to construct the image. One of the reasons we chose to share this excerpt was to highlight one of the many times our students have decided to bring in the academic language of their current unit of study. Tameka's question about Kendrae's Ls allowed the teacher to seize a short moment of connection to the unit, and it allowed the students to explore the mathematics of the image further. Students were able to flexibly consider possible rigid transformations and determine their applicability on the basis of the number of conditions (shape and/or color).

FINAL THOUGHTS

Spatial sense is an important aspect of our daily life and contributes to students' success in other areas of mathematics (Cheng and Mix 2013). Although we have shared specific details about kindergarten to grade 6 experiences with Quick Blocks, we have found that middle school, high school, and college students found the activity challenging and enjoyable. In particular, using the activity helped our high school students begin class with something everyone found challenging yet allowed students who struggle with elements of number to continue their development in an accessible way. Number sense and subitizing is one area that can be easily coupled with experiences that develop students' spatial sense. Through tasks such as Quick Blocks, students can further develop such ideas as unitizing, composing and decomposing of number and shape, orientation of objects in space, and mental and kinesthetic manipulation of imagery.

Quick Blocks has been one way in which all students can share their mathematical voice in our classrooms. It has helped our classroom communities develop norms conducive to thoughtful mathematical discourse for learning. We continue to consider and create new variations of our Quick Block activities. We invite the mathematics teaching community to use them with students and provide further thoughts and comments.

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